

ZL-125**May-2014****B.C.A. Sem.-II****CC-111 : Mathematical Foundation of Computer Science****Time : 3 Hours]****[Max. Marks : 70**

1. (a) Define group. Let $G = \{2^x : x \in \mathbb{Z}\}$, then prove that (G, \cdot) is a group. Is it Abelian group ? Justify. 7

OR

Define order of an element in the group. Find the order of each element of the group. $(\mathbb{Z}_7, +_7)$.

- (b) (1) Define cyclic group. Find a cyclic subgroup of $(\mathbb{Z}, +)$ taking 2 as a generator. 3
 (2) Let $G = \{1, a, b, ab\}$ such that $a^2 = 1, b^2 = 1, ba = ab$. Construct the multiplication table for the group. Also, find a sub-group of G . 4

OR

- (1) State the Lagrange's theorem and explain with an example. 3
 (2) Let $G = \{x \in \mathbb{R} \mid x > 1\}$ be the set of all real numbers greater than 1. 4

Define $x * y = xy - x - y + 2$, for x, y in G .

- (i) Show that the associative law holds for $*$.
 (ii) Show that 2 is the identity element for the operation $*$.

2. (a) Draw the Hasse diagram of partially ordered set $(P(X), \subseteq)$, where $X = \{1, 2, 3\}$. Answer the following : 7

- (i) Find maximal and minimal elements of $P(X)$.
 (ii) Find GLB of a subset $A = \{\{1\}, \{2\}, \{3\}\}$
 (iii) Find LUB of a subset $B = \{\{1, 2\}, \{2, 3\}, \{3, 1\}\}$
 (iv) Is $P(X)$ chain ? Why ?

OR

Let $X = \{1, 2, \dots, 8\}$ and relation $R = \{(x, y) \mid x - y \text{ is divisible by } 4\}$. Write members of R and show that R is an equivalence relation. Draw the graph of R and write relation matrix.

- (b) (1) Define Binary relation, domain and range of relation with an example. **3**
 (2) Define partition a set and give an example of a set with atleast three distinct partitions. **4**

OR

- (1) Draw the Hasse diagram of the following posets under divisibility relation : **3**
 (i) $\{1, 2, 3\}$
 (ii) $\{3, 5, 17\}$
 (2) Draw the digraph of the relation R on $A = \{1, 2, 3, 5, 6, 7\}$ as follows :
 Let $x R y$ whenever y is divisible by x. Is R equivalence relation ? Is R partial ordering ? **4**

3. (a) Define lattice with an example and list the important properties of the lattice. **7**

OR

For partially order set $\langle S_{12}, D \rangle$, $\langle S_{15}, D \rangle$ and $\langle P, D \rangle$ where $P = \langle 1, 3, 5, 7 \rangle$

- (i) Find which of these are lattices ?
 (ii) Find minimal and maximal elements in $\langle P, D \rangle$
 (iii) Are $\langle S_{12}, D \rangle$ and $\langle S_{15}, D \rangle$ distributive lattices ?
 (iv) is $\langle S_{15}, D \rangle$ complemented ?
 (b) (1) Define : **3**
 (i) Complemented Lattice
 (ii) Complete Lattice
 (iii) Boolean Expression
 (2) Define Sub-Boolean algebra; give an example of a Boolean algebra, and its Sub-Boolean Algebra. **4**

OR

- (1) Define Isomorphic Boolean algebra. Give an example of the Boolean algebra, which is isomorphic to $\langle P(X), \subseteq \rangle$ where $X = \{a, b, c\}$ **3**
 (2) Obtain sum of products canonical form of the following Boolean expressions. **4**
 (i) $x_1' * x_2$
 (ii) $(x_1 * x_2) \oplus x_3'$

4. (a) Define :

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- (i) Reachability of a Node
- (ii) Distance between the Nodes
- (iii) Binary trees
- (iv) Path matrix
- (v) Null graph
- (vi) Complete graph
- (vii) Bipartite graph

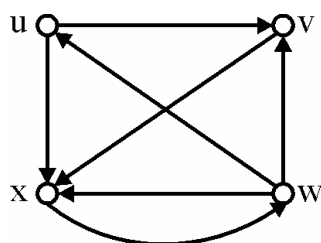
OR

Define directed tree, give other three representations of the tree given by

$((v_0(v_1(v_2))(v_3(v_4)(v_5)))(v_6(v_7(v_8))(v_9(v_{10}(v_{11}))))))$

- (b) (1) Consider the directed graph G given in the figure and answer the following questions :

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- (i) Find Adjacency matrix A of the graph.
 - (ii) Find all simple paths from u to w.
 - (iii) Find a cycle of length 2 at v.
- (2) Define nodebase of a graph and list two properties of the nodebase.

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OR

- (1) Define isomorphic graphs with an illustration.
- (2) Explain notations :
 - (i) K_n
 - (ii) $K_{m,n}$ with example

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5. Do as directed :

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- (1) Give an example of a binary relation on \mathbb{N} .
 - (2) Give an example of subgroup of $(\mathbb{R} - \{0\}, \cdot)$
 - (3) Say True/False : Identity element in a group is unique.
 - (4) Define a relation on \mathbb{N} as “ x is related to y ” means $1 \leq x + y$. Is it reflexive ? Justify.
 - (5) Find GLB (if exist) of the poset $(\{2, 3, 5, 7\}, D)$.
 - (6) Say True/False with reason : Equivalence class of an element is always non empty.
 - (7) Say True/False with reason : Every poset is a lattice.
 - (8) Define linearly order relation.
 - (9) State the absorption laws in a Boolean algebra.
 - (10) What are the meet and join operations in a Power Set Boolean Algebra ?
 - (11) Is (D_{210}, D) a Boolean algebra ? Why ?
 - (12) How many edges are there the complete graph with n -vertices ?
 - (13) Draw a complete 3-ary tree.
 - (14) Say True/False : Total degree of a graph is even number.
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